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Finite Element Model Updating for Helicopter Rotor Blade Using Genetic Algorithm

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Introduction

THE main rotor is the major subsystem in a helicopter and has considerable influence on helicopter performance, dynamics, and aeroelasticity.[1] Therefore, considerable attention is given to accurate structural dynamic modeling of the helicopter rotor blade. Typically, the helicopter rotor blade is modeled as a rotating beam using an approximate method such as the finite element method. If the natural frequencies and mode shapes predicted by the finite element model differ from test data, the mass and stiffness properties of the finite element model can be changed using a model updating method.[2] Model updating problems can be posed as optimization problems, where the difference between the available test and predicted system properties is minimized in a least-squares sense. Optimization methods such as genetic algorithms (GA) have only been used recently in model updating.[3–5] In this study we use genetic algorithms[6] for constructing a helicopter rotor blade from its frequencies.

Structural Modeling

The hingeless rotor blade is treated as a rotating cantilever beam. The free vibration of a slender, straight beam with axial force acting is governed by the equation[1]

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w}{\partial x^2}\right] + m(x)\frac{\partial^2 w}{\partial t^2} - \frac{\partial}{\partial x}\left\{\left[\int_x^R m(x)\Omega^2 x \,\mathrm{d}x\right]\frac{\partial w}{\partial x}\right\} = 0 \tag{1}$$

where $m(x)$ is the mass per unit length of the beam at section x, $I(x)$ the second moment of area about the neutral axis, E the Young's modulus, Ω the rotational speed, x the coordinate along the beam, and R the rotor blade radius. The blade is discretized into a number of beam finite elements, each with four degrees of freedom. There are two boundary nodes, and each node has deflection and slope as two degrees of freedom. Element stiffness and mass matrices for the elements are formed and assembled to obtain the global stiffness and mass matrix, respectively, and cantilever boundary conditions are applied. The natural frequencies are found by solving the general eigenvalue problem using the Jacobi method to obtain the natural frequencies:

$$\boldsymbol{K}\Phi = \omega^2 \boldsymbol{M}\Phi \tag{2}$$

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where $\boldsymbol{K}$ and $\boldsymbol{M}$ are the global stiffness and mass matrix, Φ the eigenvectors, and ω the natural frequencies. Because beam elements are used, the degree of freedom for the global stiffness and mass matrix used to calculate frequencies from the eigenvalue problem is quite small, making the process computationally efficient and therefore suitable for GA-based model updating.

Optimization Problem

The unconstrained objective function used in present formulation is

$$f = \sqrt{\sum_{i=1}^{N}\left(\frac{\omega^i_{\text{test}} - \omega^i_{\text{model}}}{\omega^i_{\text{test}}}\right)^2} \tag{3}$$

where N is the number of modes available for model updating and ω are the natural frequencies. There are $2N_e$ design variables representing the mass per unit length and stiffness for each of the N_e elements (EI_i, m_i, $i = 1, 2, \ldots, N_e$). The elements in the model are numbered from tip ($i = 1$) to root ($i = N_e$). Constraints are also applied to obtain a solution that will closely represent the original blade. They assume that the total mass and inertia of the blade are known. The total mass of the beam is given as

$$M = \int_0^R m\,\mathrm{d}r = \sum_{i=1}^{N_e}\int_{x_i}^{x_{i+1}} m\,\mathrm{d}x = \sum_{i=1}^{N_e} m(x_{i+1} - x_i) = \sum_{i=1}^{N_e} m_i l_i$$

where m_i and l_i are the mass per unit length and length of ith element, respectively. The first equality constraint can be written as

$$h_1 = \sqrt{[(M - m^0)/m^0]^2} \tag{4}$$

where m^0 is the baseline total mass of the beam obtained from the design requirements. The preceding constraint is a measure of the absolute value of the change in blade mass from the design requirements.

The second constraint applied on blade flapping inertia ensures that the Lock number ($\gamma = \rho_a acR^4/I_b$) of the blade, a key parameter in helicopter rotor design, does not change from the starting value. Here ρ_a is the density of air, a the lift curve slope, and c the blade chord. The Lock number is the ratio of aerodynamic forces acting on the blade to the inertial forces. This constraint also ensures that the rotor has enough inertia for autorotation in case of power failure:

$$I_b = \int_0^R mr^2\,\mathrm{d}r = \sum_{i=1}^{N_e}\int_{x_i}^{x_{i+1}} m_i x^2\,\mathrm{d}x = \frac{1}{3}\sum_{i=1}^{N_e} m_i\left(x^3_{i+1} - x^3_i\right)$$

The second equality constraint can be written as

$$h_2 = \sqrt{\left[\left(I_b - I_b^0\right)/I_b^0\right]^2} \tag{5}$$

where I_b^0 is the baseline mass moment inertia of the rotor blade obtained from design requirements. The preceding constraint is a measure of the absolute value of the change in blade inertia from the design requirements. Move limits are given with a variation of 20–30% from the baseline values. The move limits are needed by the GA to create a starting design population. By applying suitable penalty parameters, the preceding two equality constraints h_1, h_2 are added to the objective function f (Ref. 6). Now the objective function becomes

$$J = f + r(h_1 + h_2) \tag{6}$$

The parameter r is a scalar penalty parameter and is selected as 10 to ensure that the objective function is penalized for constraint violation.

Numerical Results

The primary objective of this work is to construct a helicopter rotor blade from its natural frequencies. The GA-based algorithm just discussed is validated by giving it simulated test frequencies from the finite element model and observing how closely it is able to construct the actual blade. The following blade properties similar to BO105 rotor are used[7]: radius of rotor blade $R = 4.94$ m, rotor speed $\Omega = 40.105$ rad/s, and mass per unit length $m_0 = 6.46$ kg/m. The nondimensional flap bending stiffness and mass per unit length for the uniform blade are obtained as $EI/m_0\Omega^2R^4 = 0.0108$ and $m/m_0 = 1.0$, respectively. For numerical results 10 finite elements and 10 modes are used.

In the GA the starting population is 20 with 10 bit representation of each design variable, maximum number of generations is 50, crossover probability is 0.6, and mutation probability is 0.04. In all cases the finite element model is used to find the value of the objective function. The GA code uses single point crossover, tournament selection, and elitism. To evaluate the GA-based algorithm, studies are first conducted on a uniform blade and a blade with linearly varying mass and stiffness along the span. Then some results for limited simulated test data using only the first four modes are obtained.

For the uniform blade the frequencies obtained from the finite element formulation are used to construct the element mass and stiffness using the genetic algorithm. The comparision of actual and predicted model mass and stiffness are shown in Table 1. The variation in mass and stiffness of each of the element in the predicted model range from 0–2% of the exact values. The natural frequency variation for the predicted blade range from 0–0.02% of the actual values. Next, a blade with linear variation in mass and stiffness is considered. The mass and stiffness decrease linearly from the root to the tip. At the tip the mass and stiffness are at 75% of the values at the root. The root stiffness and mass are $EI/m_0\Omega^2R^4 = 0.0108$ and $m/m_0 = 1.0$, respectively. Frequencies obtained from the finite element formulation are used to construct the beam element parameters using the genetic algorithm. The predicted mass and stiffness range from 0 to 2.5% of the actual values, as shown in Table 2. The frequencies of the updated model range from 0 to 0.29% of the actual values. The results show that both the uniform and the linear blade are constructed successfully from the frequency measurement by the GA-based approach.

Sometimes, the frequencies of the first 10 flap bending modes might not be available. So we consider a case where only the first four modal frequencies are available, for both a nonrotating and rotating blade. Let the first four frequencies obtained from experiment be 5, 7, 10, and 15% less than the first four natural frequencies predicted by a finite element model of the rotor crudely approximated as a uniform blade ($EI/m_0\Omega^2R^4 = 0.0108, m/m_0 = 1.0$). The element mass and stiffness distribution for the simulated test data predicted by the GA-based algorithm are shown in Tables 3 and 4, for a nonrotating and rotating blade, respectively. The predicted models have reduced stiffness along the entire blade span and almost the same mass distribution as the baseline uniform blade. Because the rotating beam has centrifugal stiffening, the stiffness reduction needed by the GA is quite large, and the move limits were placed at 30% from the baseline blade in this case. For the nonrotating blade the difference between the test and model frequencies is reduced for the first, second, third, and fourth mode from 5, 7, 10, and 15% to 0.31, 1.58, 1.31, and 4.67%, respectively. For the rotating blade the difference between the test and model frequencies are reduced for the first, second, third, and fourth mode from 5, 7, 10, and 15% to 3.7, 0.61, 0.84, and 5.46%, respectively. The updated finite element model is therefore considerably improved compared to the original model. If the first few mode frequencies, blade mass and inertia of a blade are available, the algorithm developed in this study can be used to predict the blade properties.

Table 1 Actual and predicted mass and stiffness for uniform blade

Element	Actual stiffness	Predicted stiffness	Actual mass	Predicted mass
1	0.01080	0.01083 (−0.3%)	1.0000	1.0099 (−1.0%)
2	0.01080	0.01065 (1.4%)	1.0000	0.9848 (1.5%)
3	0.01080	0.01067 (1.2%)	1.0000	0.9854 (1.5%)
4	0.01080	0.01089 (−0.8%)	1.0000	1.0114 (−1.1%)
5	0.01080	0.01076 (0.4%)	1.0000	0.9922 (0.8%)
6	0.01080	0.01085 (−0.5%)	1.0000	0.9958 (0.4%)
7	0.01080	0.01076 (0.4%)	1.0000	0.9858 (0.4%)
8	0.01080	0.01079 (0.1%)	1.0000	0.9973 (0.3%)
9	0.01080	0.01080 (0.0%)	1.0000	1.0040 (−0.4%)
10	0.01080	0.01081 (−0.1%)	1.0000	1.0200 (−2.0%)

Table 2 Actual and predicted mass and stiffness for linear blade

Element	Actual stiffness	Predicted stiffness	Actual mass	Predicted mass
1	0.008235	0.008224 (0.1%)	0.7625	0.7623 (0.0%)
2	0.008505	0.008616 (−1.4%)	0.7875	0.7940 (−0.8%)
3	0.008755	0.008694 (0.8%)	0.8125	0.8002 (1.5%)
4	0.009045	0.008935 (1.2%)	0.8375	0.8169 (2.5%)
5	0.009315	0.009451 (−1.5%)	0.8625	0.8636 (−0.1%)
6	0.009585	0.009480 (1.0%)	0.8875	0.8785 (1.0%)
7	0.009855	0.009700 (1.5%)	0.9125	0.9051 (0.8%)
8	0.010125	0.010186 (−0.7%)	0.9375	0.9439 (−0.7%)
9	0.010395	0.010337 (0.5%)	0.9625	0.9715 (−0.9%)
10	0.010665	0.010547 (1.1%)	0.9875	0.9865 (0.1%)

Table 3 Predicted mass and stiffness for nonrotating blade with simulated test data

Element	Predicted stiffness	Predicted mass
1	0.00823	1.0236
2	0.00810	0.9193
3	0.00820	1.0485
4	0.00985	0.9154
5	0.00824	1.0380
6	0.00814	1.0473
7	0.00935	1.0364
8	0.00815	1.0281
9	0.01050	0.9676
10	0.01090	1.0410

Table 4 Predicted mass and stiffness for rotating blade with simulated test data

Element	Predicted stiffness	Predicted mass
1	0.00814	1.0486
2	0.00807	0.9423
3	0.00820	1.0113
4	0.00803	1.0298
5	0.00802	1.0266
6	0.00804	1.0262
7	0.00810	0.9987
8	0.00859	0.9825
9	0.00856	0.9825
10	0.00851	0.9977

Conclusions

In this study we address the problem of constructing a hingeless helicopter rotor blade using a genetic-algorithm-based approach. The model updating problem is posed as an optimization problem. Constraints on total mass and flap inertia are included in the objective function using suitable penalty parameters. Move limits of 20–30% are imposed on the element mass and stiffness. Ten finite elements of equal length are used for modeling the blade, and 10 modes are used for reconstructing the blade. Design variables are the finite element mass and stiffness. The following conclusions are drawn from this study:

1) When the algorithm is applied for rotor blades with uniform and linear stiffness and mass variations, the beam stiffness and mass parameters are predicted within 2.5%. The frequencies of the blade are predicted within 0.29% by the updated model.

2) A situation is considered where simulated test data for only the first four modes are available. In this case the algorithm constructs a blade by matching the model frequencies with the test data

closely. The updated model is considerably improved compared to the original finite element model.

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Golinski's Speed Reducer Problem Revisited

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Introduction

GOLINSKI'S[1,2] speed reducer problem is one of the most well-studied problems of the NASA Langley Multidisciplinary Design Optimization (MDO) Test Suite. Golinski modeled the speed reducer with an aim to minimize its weight while satisfying a number of constraints imposed by gear and shaft design practices. Since then, many researchers, for example, Rao,[3] Li and Papalambros,[4] Kuang et al.,[5] and Azarm and Li[6] have reported solutions of this problem. However, the solutions reported by all of the researchers just mentioned are not feasible, including the one that appears in the MDO Test Suite itself, obtained using the constrained minimizer CONMIN.[7] The present Note presents the best-known feasible solution to this problem and provides a comparison of this solution with those of Refs. 3–7. The details of the swarm algorithm are described in the Appendix. The notation and sequence of constraints used in the present Note are the same as the problem description in the NASA Langley MDO Test Suite.

Problem Statement

The objective is to find the minimum gearbox volume f (and hence, its minimum weight), subject to several constraints. There are seven design variables: width of the gear face x_1, teeth module x_2, number of pinion teeth x_3, shaft 1 length between bearings x_4, shaft 2 length between bearings x_5, diameter of shaft 1 x_6, diameter of shaft 2 x_7. These objectives lead to the following constrained optimization problem.

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Minimize:

$$f(\boldsymbol{x}) = 0.7854x_1x_2^2\left(3.3333x_3^2 + 14.9334x_3 - 43.0934\right) - 1.5079x_1\left(x_6^2 + x_7^2\right) + 7.477\left(x_6^3 + x_7^3\right) + 0.7854\left(x_4x_6^2 + x_5x_7^2\right)$$

Subject to (constraints in right-hand column)

$$27x_1^{-1}x_2^{-2}x_3^{-1} \le 1 \qquad G_1$$
$$397.5x_1^{-1}x_2^{-2}x_3^{-2} \le 1 \qquad G_2$$
$$1.93x_2^{-1}x_3^{-1}x_4^3x_6^{-4} \le 1 \qquad G_3$$
$$1.93x_2^{-1}x_3^{-1}x_5^3x_7^{-4} \le 1 \qquad G_4$$
$$\left[\left(745x_4x_2^{-1}x_3^{-1}\right)^2 + 16.9\times 10^6\right]^{\frac{1}{2}} \Big/ \left[110.0x_6^3\right] \le 1 \qquad G_5$$
$$\left[\left(745x_5x_2^{-1}x_3^{-1}\right)^2 + 157.5\times 10^6\right]^{\frac{1}{2}} \Big/ \left[85.0x_7^3\right] \le 1 \qquad G_6$$
$$x_2x_3/40 \le 1.0 \qquad G_7$$
$$5x_2/x_1 \le 1.0 \qquad G_8$$
$$x_1/12x_2 \le 1.0 \qquad G_9$$
$$(1.5x_6 + 1.9)x_4^{-1} \le 1 \qquad G_{24}$$
$$(1.1x_7 + 1.9)x_5^{-1} \le 1 \qquad G_{25}$$

Constraints G_{10} and G_{11} are the side constraints of x_1, constraints G_{12} and G_{13} are the side constraints of x_2, constraints G_{14} and G_{15} are the side constraints of x_3, constraints G_{16} and G_{17} are the side constraints of x_4, constraints G_{18} and G_{19} are the side constraints of x_5, constraints G_{20} and G_{21} are the side constraints of x_6, and constraints G_{22} and G_{23} are the side constraints of x_7. The variable bounds for the problem are as follows: $2.6 \le x_1 \le 3.6$, $0.7 \le x_2 \le 0.8$, $17 \le x_3 \le 28$ (integer values), $7.3 \le x_4 \le 8.3$, $7.3 \le x_5 \le 8.3$, $2.9 \le x_6 \le 3.9$, and $5.0 \le x_7 \le 5.5$.

Discussion

From Table 1 it is clear that the solutions reported by Rao,[3] Li and Papalambros,[4] and Azarm and Li[6] violate constraint G_5 and G_{25}. The result reported by Kuang et al.[5] violates G_6, and that using CONMIN at the MDO test suite violates G_5, G_6, and G_{25}. Although for some solutions the amount of constraint violation is marginal, the solutions are still infeasible form a mathematical point of view.

Swarm algorithms have been applied successfully to single-objective unconstrained and constrained optimization problems by Ray and Liew[8] and to engineering design optimization problems by Ray and Saini.[9] A swarm size of 70 has been used in this study, and the results are after 70,000 function evaluations. Results of five successive runs are presented in Table 1 along with their objective function values. The best solution obtained using the swarm algorithm is compared with other reported results in Table 2.

Conclusions

This Note provides a comparison between the results reported by various sources to Golinski's speed reducer problem. It is interesting to take note that several of these reported solutions are infeasible. A swarm algorithm has been used in this study to solve the nonlinear constrained minimization problem and arrive at the best known feasible solution.

Appendix: Swarm Algorithm

A general constrained optimization problem (in the minimization sense) is presented as follows.

Minimize:

$$f(x)$$

Subject to:

$$g_i(\boldsymbol{x}) \ge a_i, \qquad i = 1, 2, \ldots, q$$

$$h_j(\mathbf{x}) = b_j, \qquad j = 1, 2, \ldots, r$$

where there are q inequality and r equality constraints, $\boldsymbol{x} = [x_1\ x_2\ \ldots\ x_n]$ is the vector of n design variables, and a_i and b_j